1. The geometric mean of six numbers is approximately 14.508. (The geometric mean of n numbers is the nth root of the product.) If three of the numbers are doubled and the other three are tripled, what will the approximate geometric mean of the resulting six numbers be?

   (a) 72.539
   (b) 35.537
   (c) 87.046
   (d) 36.270
   (e) Cannot be determined from the information given.

2. The angle of elevation from the right edge of the top of a 50 foot building to a skyscraper located on the right side of the building is 59°. The angle of elevation from street level at the right edge of the 50 foot building to the skyscraper is 62°. What is the shortest distance between the tops of the two buildings?

   (a) 434 feet.
   (b) 499 feet.
   (c) 449 feet.
   (d) 384 feet.
   (e) 484 feet.

3. What is the approximate difference between the area of an octagon circumscribed about a circle of radius 1 and the area of a hexagon inscribed in a circle of radius 1?

   (a) 0.7156
   (b) 0.1721
   (c) 0.5435
   (d) 0.6361
   (e) 0.3224

4. The equation $16x^2 - 64x = -9y^2 - 54y - 1$ represents which of the following?

   (a) A circle
   (b) A hyperbola
   (c) A parabola
   (d) An ellipse
   (e) An epicycloid

5. Let U, the universal set, be the set of natural numbers. Given the following sets $A = \{2, 5, 8, 14, 23, 68\}, B = \{5, 6, 11, 14, 35, 51, 68\}, C = \{5, 6, 8, 31, 51\}$ which of the following describes the set {2,6,23,51}? (Note: C' represents the complement of C)

   (a) $A \cap B \cap C'$
   (b) $(A \cap B') \cup (A \cap C')$
   (c) $(A \cap B') \cap (A \cap C')$
   (d) $(A - (B \cup C)) \cup ((B \cap C) - A)$
   (e) $(A' - (B \cap C))$

6. There are 10 students in a fifth year Latin class. The class average on the third test was 84. Jack's score was 50 and Jill's score was 70. The teacher wants to exclude these two scores and find the average for everyone else in the class. What is the correct average for everyone else in the class?

   (a) 50
   (b) 70
   (c) 84
   (d) 90
   (e) 72

7. Simplify $i^{1,000,000,001}$.

   (a) 1
   (b) i
   (c) $-i$
   (d) -1
   (e) Cannot be determined from the information given.

8. Let N be the smallest natural number which is the square of one natural number and the fifth power of a different natural number. How many digits does N have?

   (a) 2
   (b) 3
   (c) 4
   (d) 5
   (e) 6

9. The radius of a circular pond is 30 feet, the radius of a circular lake is 700 feet, and the center of the lake is 600 feet east and 800 feet north of the center of the pond. If a baby duck walks from the edge of the pond to the edge of the lake, then what is the shortest distance that she must walk?

   (a) 260 feet
   (b) 270 feet
   (c) 280 feet
   (d) 290 feet
   (e) 300 feet

10. If $\tan(x + y) = 33$ and $\tan(x) = 3$, find $\tan(y)$.

    (a) 30
    (b) 3
    (c) .6
    (d) .3
    (e) .33

11. Suppose that A, B, and C are collinear points such that B is between A and C. If $AB = \frac{5}{8}AC$ and $AB = 15$, find BC.

    (a) 9
    (b) $\frac{75}{8}$
    (c) 24
    (d) 33
    (e) 17

12. In Morse code, symbols are represented by variable length sequences of dots and dashes. (For example, $A = \cdot -$, $1 = \cdot - - - -$, and $? = \cdot \cdot - - \cdot \cdot$.) How many different symbols can be represented by sequences of six or fewer dots and dashes?

(a) 42
(b) 126
(c) 128
(d) 254
(e) 1024

13. A carpenter makes 3-legged stools and 4-legged stools. They have the same kind of legs and the same kind of tops, but the 4-legged stools have 4 crossbars between the legs and the 3-legged stools have no crossbars. If the carpenter has 68 crossbars and 25 tops, how many legs should she make in order to exactly use up all of the crossbars and tops?

(a) $\sqrt{17}$
(b) 25
(c) 68
(d) 92
(e) 104

14. Suppose that to make a dunce hat you take a circle and cut out a sector with angle 135 degrees, throw it away, and then tape the two resulting sides together. But then the person for whom you made the dunce hat fills it with water and dumps it on your head. If the circle had radius 6cm, approximately how much water is dumped on your head?

(a) $(1/3)\pi 6^3\text{cm}^3$
(b) $(1/3)\pi(3.75^2)(4.6837)\text{cm}^3$
(c) $\pi(3.75^2)(4.6837)\text{cm}^3$
(d) $(1/3)\pi(3.75^2)6\text{cm}^3$
(e) $\pi(3.75^2)6\text{cm}^3$

15. At a certain time of day, a 5 foot tall woman has a 10 foot shadow. What is the angle of elevation to the sun from the tip of the woman's shadow?

   (a) $\cos 2$
   (b) $\sin 0.5$
   (c) $\arcsin 0.5$
   (d) 0.5
   (e) $\arctan 0.5$

16. An small bug wanders around on a handout from math class. The bug's path follows the ellipse that your teacher drew. The ellipse crosses the x-axis at 2 and -2 and the y-axis at 3 and -3. You spilled coke on the paper and it has covered the drawing for $y \geq 2$. At what positive x-value will the bug encounter the coke?

   (a) 2
   (b) $\dfrac{2\sqrt{5}}{3}$
   (c) $\dfrac{3\sqrt{13}}{2}$
   (d) 0
   (e) $\dfrac{2}{9}$

17. In the triangle below let $\alpha = x^\circ$, $\beta = (x+20)^\circ$ and $\gamma = (210-3x)^\circ$. Find the measure of each angle.

   (a) $\alpha = 60^\circ$, $\beta = 70^\circ$, $\gamma = 50^\circ$
   (b) $\alpha = 50^\circ$, $\beta = 60^\circ$, $\gamma = 70^\circ$
   (c) $\alpha = 50^\circ$, $\beta = 70^\circ$, $\gamma = 60^\circ$
   (d) $\alpha = 60^\circ$, $\beta = 50^\circ$, $\gamma = 70^\circ$
   (e) $\alpha = 70^\circ$, $\beta = 60^\circ$, $\gamma = 50^\circ$

18. Find the sum of all solutions of $|x^2 - 3x| = -4x + 6$

    (a) 6
    (b) -2
    (c) 8
    (d) 0
    (e) 4

19. If the length and width of a rectangle are doubled, then the area of that rectangle is multiplied by what factor?

    (a) 2
    (b) 4
    (c) 6
    (d) 8
    (e) 10

20. The diameter of the circle shown is 8 inches. What is the perimeter of the inscribed square $ABCD$?

    (a) $4\sqrt{2}$
    (b) 8π
    (c) 4π
    (d) 64π
    (e) $16\sqrt{2}$

21. The empty set, $\emptyset$, is a subset of which of the following sets?

    A is the set of real numbers

    B is the set of whole numbers

    C is the set of integers

    (a) A
    (b) B
    (c) C
    (d) A, B, and C
    (e) none of these sets

22. When an airplane leaves the runway, its angle of climb is 18° and its speed is 275 feet per second. Find the plane's approximate altitude after 1 minute.

    (a) 5099 feet
    (b) 15692 feet
    (c) 85 feet
    (d) 262 feet
    (e) 5361 feet

23. What is the domain of the function $f(x) = \ln(x^3 - 3x^2 - 2x + 6)$?

    (a) $\left(-\infty, -\sqrt{2}\right) \cup \left(\sqrt{2}, 3\right)$
    (b) $\left(-\sqrt{2}, \sqrt{2}\right) \cup (3, \infty)$
    (c) $\left(-\infty, \sqrt{2}\right) \cup (3, \infty)$
    (d) $\left(-\infty, -\sqrt{2}\right) \cup (3, \infty)$
    (e) $\left(-\infty, -\sqrt{2}\right) \cup \left(\sqrt{2}, \infty\right)$

24. Consider the two curves given below.

$$\begin{aligned} y &= x + 1 \\ x^2 + y^2 &= 16 \end{aligned}$$

    The sum of the x-coordinates of the points of intersection of these two curves is

    (a) The curves do not intersect.
    (b) 1
    (c) 0
    (d) -1
    (e) $\dfrac{-1 + \sqrt{2}}{2}$

25. Let ΔDEF be an equilateral triangle with side length one. Suppose ΔABC is inscribed in ΔDEF as shown below.

    Find $\sin(\angle ABE)$.

    (a) $\dfrac{3\sqrt{21}}{14}$
    (b) 1
    (c) $\dfrac{\sqrt{3}}{2}$
    (d) $\dfrac{\sqrt{5}}{2}$
    (e) $\dfrac{3\sqrt{3}}{14}$

26. A dog, Mandy, was 20 feet ahead of her owner, David, when he started chasing after her in order to catch her. If David caught the dog in 5 seconds and ran twice as fast as Mandy ran, how far did David have to go to catch Mandy?

(a) 8 feet
(b) 10 feet
(c) 15 feet
(d) 25 feet
(e) 40 feet

27. The line passing through $(m^2+1, 5)$ and $(3, m)$ has an undefined slope for what values of m?

(a) $\sqrt{2}$
(b) 5
(c) $\sqrt{2}$ or $-\sqrt{2}$
(d) 5 or -5
(e) -5

28. The measure of the exterior angle of a regular octagon is

(a) $30°$
(b) $45°$
(c) $60°$
(d) $120°$
(e) $135°$

29. If $7^{6y} = 125$, find 7^{-y}.

(a) $\dfrac{-\sqrt[6]{125}}{7}$
(b) $\dfrac{\sqrt[6]{125}}{7}$
(c) $\dfrac{1}{5}$
(d) $\dfrac{\sqrt{5}}{5}$
(e) $\dfrac{\sqrt{5}}{125}$

30. Find the cosine of the smallest angle of a triangle that has sides 2,6, and 7.

   (a) $\dfrac{89}{84}$
   (b) 11
   (c) $\dfrac{27}{14}$
   (d) $\dfrac{27}{7}$
   (e) $\dfrac{27}{28}$

31. If 4 coins are tossed in the air, what is the probability that at least one will turn up heads?

   (a) $\dfrac{1}{16}$
   (b) $\dfrac{1}{4}$
   (c) $\dfrac{1}{2}$
   (d) $\dfrac{3}{4}$
   (e) $\dfrac{15}{16}$

32. In the figure below, a square is inscribed in a circle. The area of the shaded region is $100(\pi - 2)$. Find the diameter of the circle.

   (a) 10
   (b) $10\sqrt{\pi - 2}$
   (c) $\sqrt{\dfrac{200(\pi - 2)}{2\pi - 1}}$
   (d) $\sqrt{\dfrac{100(\pi - 2)}{\pi}}$
   (e) 20

33. Consider the numbers below.

$$\begin{array}{ccccccccccc} 20 & 1 & 7 & 19 & 5 & 12 & 2 & 13 & 4 & 18 & 8 \\ & 17 & 16 & 9 & 10 & 3 & 6 & 11 & 14 & 15 & \end{array}$$

Find the sum of the values of the numbers that lie at or below Q_1 (the first quartile).

(a) 15
(b) 55
(c) 120
(d) 210
(e) 0

34. Find the area of a circle that has the points (2,2) and (-4,-6) as the endpoints of a diameter.

(a) 10π
(b) 5π
(c) 25π
(d) 100π
(e) none of the above.

35. A social club charters a bus at a cost of \$900 to take a group of members on an excursion to Atlantic City. At the last minute five people in the group decided not to go. This raises the transportation cost per person by \$2. How many people originally intended to take the trip?

(a) 38
(b) 40
(c) 45
(d) 50
(e) 55

36. A jeweler has three small solid spheres made of gold, of radius 2 inches, 3 inches, and 4 inches. He decides to melt these down and make just one sphere out of them. What is the radius of this larger sphere?

(a) 4.5
(b) $\sqrt[3]{90}$
(c) $\sqrt[3]{99}$
(d) 4.3
(e) $\sqrt[3]{100}$

37. In the figure, $\angle ABC$ makes an angle of 125° with the floor and l_1 and l_2 are parallel. Determine the measure of $\angle DFE$.

(a) 45°
(b) 35°
(c) 30°
(d) 25°
(e) 55°

38. Simplify the expression $\cot(-x)\cos(-x) + \sin(-x)$.

(a) 0
(b) $\cot(-x)$
(c) $2\sin(-x)$
(d) $\tan x$
(e) $-\csc x$

39. The statement "If I do not study, then I will fail the mathematics test" is logically equivalent to which of the following statements.

(a) If I fail the mathematics test, then I do not study.
(b) I do not study, and I fail the mathematics test.
(c) I study, or I fail the mathematics test.
(d) If I study, then I do not fail the mathematics test.
(e) Studying is a sufficient condition for my not failing the mathematics test.

40. Which of the following can **not** be factored into two nonconstant polynomials with only real numbers as coefficients?

I. $x^2 + 81$
II. $x^3 + 81$
III. $x^4 + 81$

(a) I only
(b) I and II only
(c) I and III only
(d) II only
(e) I, II, and III

41. One thousand unit cubes are fastened together to form a large cube with edge length 10 units; this large cube is painted and then separated into the original cubes. How many of these unit cubes will have at least one face painted?

(a) 600
(b) 520
(c) 488
(d) 480
(e) 400

42. If $x \neq 0$ and $x \neq 4$ and $y \neq 0$ and $y \neq 6$, then

$$\frac{2}{x} + \frac{3}{y} = \frac{1}{2}$$

is equivalent to

(a) $4x + 3y = xy$
(b) $y = \dfrac{4x}{6 - y}$
(c) $\dfrac{x}{2} + \dfrac{y}{3} = 2$
(d) $\dfrac{4y}{y - 6} = x$
(e) none of these

43. Find the roots of the equation $\log_2(x^2 - x) = 1$

(a) $2, -1$
(b) $1, 0$
(c) 4
(d) $0, 2$
(e) -2

44. Determine the term that contains a^4 in the expansion of $(a - 2b^2)^7$.

(a) $280a^4b^6$
(b) $-35a^4b^3$
(c) $21a^4b^7$
(d) $35a^4b^6$
(e) none of these

45. Consider the following three statements.

I. $\sqrt{x^2+y^2} = x + y$

II. $\dfrac{2+\sqrt{x^2-y^2}}{2} = 1 + \sqrt{x^2-y^2}$

III. $\dfrac{1}{x+y} = \dfrac{1}{x} + \dfrac{1}{y}$

Which statement(s) is(are) true for all real numbers x and y?

(a) I only
(b) II only
(c) III only
(d) I and II only
(e) none of these