1. What is the volume of a cube inscribed in a sphere of radius 3?

   (A) $18\sqrt{2}$

   (B) $18\sqrt{3}$

   (C) $21\sqrt{2}$

   (D) $24\sqrt{3}$

   (E) $30\sqrt{6}$

2. Suppose there are 3 different biology books, 4 different english books, and 6 different math books and all the books need to be placed on a bookshelf. How many different ways can this be done if all the books of the same subject must be placed on the shelf together?

   (A) 20736

   (B) 34560

   (C) 103680

   (D) 311040

   (E) 622080

3. Suppose that the following holds for all x

$$10 \cdot g(5 - x) + 2 \cdot g(x) = x^3$$

   where x is a real number. What is $g(1)$?

   (A) $\frac{1}{42}$ (B) $\frac{319}{48}$ (C) $\frac{49}{8}$ (D) $\frac{511}{32}$ (E) $\frac{160}{3}$

4. Given that $\log_c 27 = 4$ and $\log_d 3 = 5$, find $\log_d c$.

   (A) $\frac{4}{15}$

   (B) $\frac{4}{5}$

   (C) 2

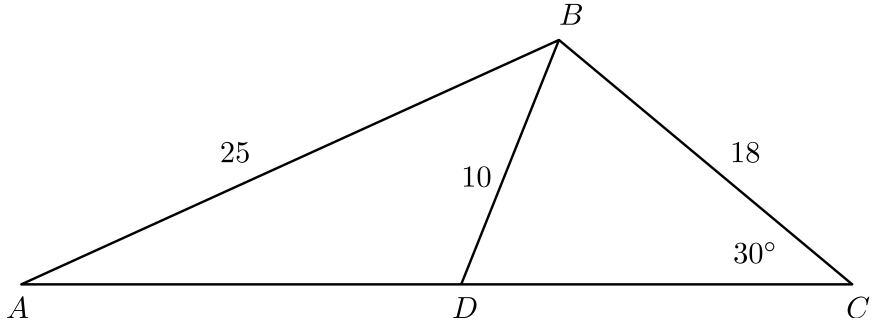
   (D) $\frac{15}{4}$

   (E) $\frac{64}{5}$

5. Sansa and Arya make four wagers over the course of a family vacation and the value of the wager triples each time. Sansa wins the first and last wager while Arya wins the second and third wager. If Sansa wins a total of \$20, what was the value of the original wager?

(A) \$ 3 (B) \$ 1.43 (C) \$ 1.25 (D) \$ 0.75 (E) \$ 0.50

6. Consider the figure below (not drawn to scale). Given that $\overline{AB} = 25$, $\overline{BD} = 10$, $\overline{BC} = 18$, and $\angle BCD = 30°$, find $\angle BDC$ rounded to the nearest degree.



(A) 59° (B) 62° (C) 64° (D) 68° (E) 71°

7. Evaluate $\lim_{x \to 1} \dfrac{x^{500} - 1}{x - 1}$, if it exists.

(A) 1 (B) 499 (C) 500 (D) 501 (E) This limit does not exist.

8. Suppose you have 8 bags of Pretzel M&M's and 6 bags of Peanut M&M'S in your back pack. If you randomly select 5 bags without replacing the bags, what is the probability of selecting 4 bags of Pretzel M&M's and 1 bag of Peanut M&M's?

(A) 0.08 (B) 0.21 (C) 0.28 (D) 0.35 (E) 0.42

9. The polynomial $12x^2 - x - 6$ factors as

(A) $(4x + 3)(3x - 2)$

(B) $(4x - 3)(3x + 2)$

(C) $(2x + 3)(6x - 2)$

(D) $(2x - 3)(6x + 2)$

(E) $(12x + 3)(x - 2)$

10. What is the remainder when $x^3 + 5x + 1$ is divided by $x + 3$?

    (A) 43

    (B) 25

    (C) 0

    (D) -5

    (E) -41

11. Which of these lines is tangent to the circle $x^2 + y^2 = 25$?

    (A) $3x - 4y = 0$

    (B) $3x - 4y = 25$

    (C) $3x - 4y = 16$

    (D) $3x - 4y = 9$

    (E) $3x - 4y = -9$

12. Consider the functions f and g defined by $f(x) = 10x^2 + 7x$ and $g(x) = \dfrac{x^3}{8}$. Which of the following statements is true?

    (A) There is a positive real number M such that for every real number $x > M$ we have $f(x) > g(x)$ and $f(-x) > g(-x)$.

    (B) There is a positive real number M such that for every real number $x > M$ we have $f(x) > g(x)$ and $f(-x) < g(-x)$.

    (C) There is a positive real number M such that for every real number $x > M$ we have $f(x) < g(x)$ and $f(-x) > g(-x)$.

    (D) There is a positive real number M such that for every real number $x > M$ we have $f(x) < g(x)$ and $f(-x) < g(-x)$.

    (E) None of these.

13. How many ways can you go up 7 steps if you go one or two at a time (upward only).

    (A) 7

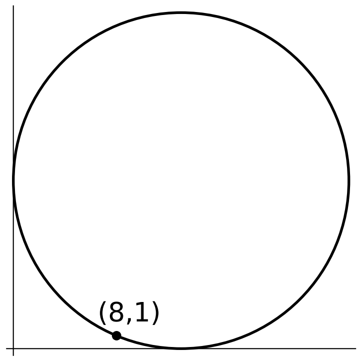
    (B) 10

    (C) 17

    (D) 19

    (E) 21

14. What is the radius of the circle that is tangent to the coordinate axes and contains the point $(8, 1)$? See the figure below.



(A) 5

(B) 7

(C) 8

(D) 13

(E) 65

15. Determine the negation of the statement, "Everybody trusts somebody."

(A) "Somebody trusts everybody."
(B) "Nobody trusts somebody."
(C) "Nobody trusts everybody."
(D) "Everybody trusts everybody."
(E) "There is somebody who trusts nobody."

16. For what values of a does the curve $y = ax^2 + 3x + 1$ intersect the line $y = -2x + 3$ in distinct two points?

(A) $a \geq \dfrac{-25}{8}$ and $a > 0$.

(B) $a > \dfrac{-25}{8}$

(C) $a \geq \dfrac{-25}{8}$

(D) $a > \dfrac{-25}{8}$ and $a \neq 0$.

(E) None of the above.

17. Five people are selected at random. What is the probability that at least two of them share a birthday? Round your final answer to four decimal places. (Ignore February 29, and assume each year has 365 days.)

(A) 0.0109

(B) 0.0271

(C) 0.0405

(D) 0.9729

(E) None of the above.

18. Which of the following has the **smallest** area?

(A) A rectangle with side lengths 4 and 8.

(B) A square with a side length of 6.

(C) A right triangle whose height is 10 and whose hypotenuse is 12.

(D) A circle with a radius of length 3.25.

(E) A parallelogram with a base of length 7 and a height of length 5.

19. If $|a| = 3,\ |b| = 2,$ and $|a+b| = a+b$, then $a-b$ equals which of the following?

(A) 1 (B) -5 (C) 1 or 5 (D) 1 or -5 (E) None of the above.

20. A corn silo is constructed by attaching a hemisphere to the top of a right circular cylinder. If the cylinder is 20 feet high and the volume of the silo is 500π cubic feet, which of the following is the common radius (rounded to two decimal places) of the cylinder and hemisphere?

(A) 4.23 feet

(B) 4.32 feet

(C) 4.40 feet

(D) 4.65 feet

(E) 4.81 feet

21. If $f(2x) = \dfrac{2}{2+x}$ for all $x > 0$, then $2f(x) =$

(A) $\dfrac{2}{1+x}$ (B) $\dfrac{2}{2+x}$ (C) $\dfrac{4}{1+x}$ (D) $\dfrac{4}{2+x}$ (E) $\dfrac{8}{4+x}$

22. Which of the following have the same graph?

I. $y = x - 2$ II. $y = \dfrac{x^2 - 4}{x + 2}$ III. $(x + 2)y = x^2 - 4$

(A) I and II only

(B) I and III only

(C) II and III only

(D) I, II and III

(E) None of the above. All the equations have different graphs.

23. A square of perimeter 20 is inscribed in a square of perimeter 28. What is the greatest distance between a vertex of the inner square and a vertex of the outer square?

(A) $\sqrt{58}$ (B) $\dfrac{7\sqrt{5}}{2}$ (C) 8 (D) $\sqrt{65}$ (E) $5\sqrt{3}$

24. Smith begins a new job at a salary of 100,000. Smith receives a 5% raise every year until she retires. Suppose that Smith works for 35 years. Determine the total salary earned over Smith's career (nearest million).

(A) 5 (B) 6 (C) 7 (D) 8 (E) 9

25. If $f\left(\dfrac{x}{x-1}\right) = \dfrac{1}{x}$ for all $x \neq 0, 1$ and $0 < \theta < \pi/2$, then $f(\sec^2\theta) =$

(A) $\sin^2\theta$

(B) $\cos^2\theta$

(C) $\tan^2\theta$

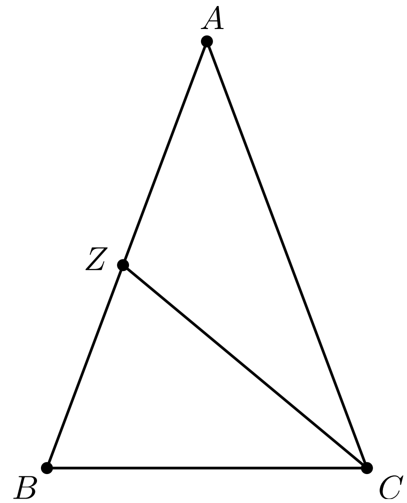
(D) $\cot^2\theta$

(E) $\csc^2\theta$

26. Find x if $x = 1 + \cfrac{1}{1 + \cfrac{1}{1 + \cfrac{1}{1 + \ddots}}}$.

(A) $\dfrac{1+\sqrt{5}}{2}$ (B) $\dfrac{1+\sqrt{6}}{2}$ (C) $1 + \dfrac{\sqrt{5}}{2}$ (D) $1 + \dfrac{\sqrt{6}}{2}$ (E) None of the above.

27. Consider the figure given below (not drawn to scale).



Suppose AB and AC have the same length and $AZ = ZC = CB$. What is $\angle ZAC$?

(A) 30° (B) 36° (C) 48° (D) 60° (E) 72°

28. Let $p > 0$. If $(p, 4)$ and $(3, p)$ lie on a line with a slope of p, then p is given by

(A) 3

(B) $1 + \sqrt{5}$

(C) $1 + \sqrt{6}$

(D) $1 + \sqrt{7}$

(E) $1 + 2\sqrt{2}$

29. In the figure below, each of the three circles in the figure are externally tangent to the other two, and each side of the triangle is tangent to two of the circles. If each circle has a radius of 3, then the perimeter of the triangle is given by

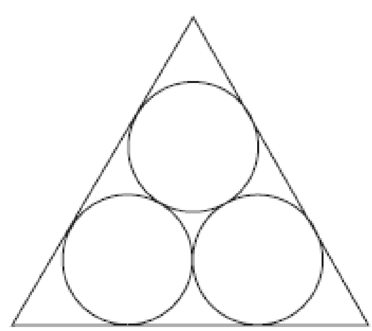
(A) $36 + 9\sqrt{2}$

(B) $36 + 6\sqrt{3}$

(C) $36 + 9\sqrt{3}$

(D) $18 + 18\sqrt{3}$

(E) 45



30. For which non-zero real number ℓ is

$$\frac{|\ell - |\ell||}{\ell}$$

a positive integer?

(A) ℓ is negative

(B) ℓ is positive

(C) ℓ is an even integer

(D) ℓ is any non-zero real number

(E) None of the above.

31. A tour boat takes three times as long to go 40 miles upstream against the current as it does to return with the current. If the boat moves at 15 miles per hour in still water, what is the rate of the current in the river?

(A) 6.5 mph (B) 7.0 mph (C) 7.5 mph (D) 8.0 mph (E) 8.5 mph

32. Suppose m is an odd integer. If the sum of the odd integers from 1 to m is 7,921, then what is m?

(A) 103 (B) 139 (C) 151 (D) 177 (E) 195

33. Suppose $\sin(\theta) + \cos(\theta) = 0.7$. Find $\sin(2\theta)$.

(A) -0.51 (B) -0.295 (C) 0 (D) 0.295 (E) 0.51

34. Which of the following equals $\sin(\arctan x)$? Here the function $\arctan x$ is defined for all real numbers x by $y = \arctan x$ if and only if $-\frac{\pi}{2} < y < \frac{\pi}{2}$ and $\tan(y) = x$.

(A) $\frac{x}{\sqrt{1+x^2}}$ (B) $\frac{1}{\sqrt{1+x^2}}$ (C) $\sqrt{x^2+1}$ (D) $\frac{\sqrt{x^2+1}}{x}$ (E) None of the above.

35. Consider the sequence given by

$$\frac{1}{4}, \frac{2}{25}, \frac{3}{100}, \frac{4}{289}, \frac{5}{676}, \ldots$$

Assuming the patterns continues in this fashion, which of the following is the 8^{th} term in the sequence?

(A) $\frac{8}{3969}$ (B) $\frac{8}{4225}$ (C) $\frac{8}{4489}$ (D) $\frac{8}{6561}$ (E) $\frac{8}{8352}$

36. The product of the roots of $p(x) = x^5 + 3x^4 - 5x^3 - 15x^2 + 4x + 12$ is given by

(A) -12 (B) 12 (C) -6 (D) 6 (E) 4

37. Solve $\cos^2(x) - \frac{\sqrt{2}}{2}\cos(x) - \frac{\sqrt{3}}{2}\cos(x) + \frac{\sqrt{6}}{4} = 0$ when $0 \leq x \leq \frac{\pi}{2}$.

(A) $\frac{\pi}{6}$ and $\frac{\pi}{3}$

(B) $\frac{\pi}{3}$ and $\frac{\pi}{2}$

(C) $\frac{\pi}{4}$ and $\frac{\pi}{3}$

(D) $\frac{\pi}{4}$ and $\frac{\pi}{6}$

(E) None of the above.

38. An urn contains 2 red marbles, 4 blue marbles, and 6 white marbles. If two marbles are chosen without replacement, find the probability that you draw two marbles of a different color.

(A) $\frac{1}{2}$ (B) $\frac{1}{3}$ (C) $\frac{2}{3}$ (D) $\frac{3}{4}$ (E) $\frac{5}{6}$

39. Three landmarks of baseball achievement are Ty Cobb's batting average of .420 in 1911, Ted Williams' .406 in 1941, Hank Aaron's .355 in 1959, and George Brett's .390 in 1980. Here are the batting average mean and standard deviations for the corresponding decades:

Decade	Mean	Standard Deviation
1910 – 1919	.266	.0371
1940 – 1949	.267	.0326
1950 – 1959	.259	.0310
1980 – 1989	.261	.0317

Which player performed better relative to his peers?

(A) Ty Cobb

(B) Ted Williams

(C) Hank Aaron

(D) George Brett

(E) a tie between Cobb and Aaron

40. A total of 36 students gathered to recycle the following items: Paper, Metal, and Glass. How many students recycled only Glass and nothing else if of the 36 students,

11 students recycled Paper

13 students recycled Metal

15 students recycled Glass

3 students recycled Paper and Metal

4 students recycled Paper and Glass

5 students recycled Metal and Glass

1 student recycled all three items.

(A) 15

(B) 8

(C) 7

(D) 10

(E) 1

41. Initially a jar contains 100 black pens and 100 red pens. Repeatedly 3 pens are removed from the jar and replaced from a pile outside the jar as follows:

Pens Removed	Replaced With
3 black	1 black
2 black and 1 red	1 black and 1 red
1 black and 2 red	2 red
3 red	1 black and 1 red

Which of the following sets of pens **could be** the contents of the jar after repeated applications of this procedure?

(A) 2 black pens

(B) 2 red pens

(C) 1 black pen

(D) 1 black pen and 1 red pen

(E) 1 red pen

42. Let $x \# y$ be defined by $x \# y = (x+1)(y+1) - 1$ for all real numbers x and y. Which of the following is false?

(A) $x \# y = y \# x$ for all real numbers x, and y

(B) $x \# (y+z) = (x \# y) + (x \# z)$ for all real numbers x, y, z

(C) $(x-1) \# (x+1) = (x \# x) - 1$ for any real number x

(D) $x \# 0 = x$ for any real number x

(E) $x \# (y \# z) = (x \# y) \# z$ for all real numbers x, y, z

43. Suppose two candles have the same height but different radii and when the candles are lit at the same time the first candle takes 3 hours to completely melt while the second candle takes 2 hours to melt. If both candles burn at a constant rate, how long after being lit was the first candle 2 times the height of the second candle?

(A) 1 hour 10 min

(B) 1 hour 20 min

(C) 1 hour 30 min

(D) 1 hour 40 min

(E) 1 hour 50 min

44. Suppose $x - 3y + z = 4$, $-x + 4y - 4z = 1$, and $2x - y + 5z = -3$. What is $x \cdot y \cdot z$?

(A) -6

(B) -5

(C) 0

(D) 5

(E) 6

45. The pairs of values (x, y) that are common solutions of the equations $y = (x+1)^2$ and $xy + y = 1$ is given by

(A) 3 real pairs

(B) 4 real pairs

(C) 4 imaginary pairs

(D) 2 real and 2 imaginary pairs

(E) 1 real and 2 imaginary pairs