Mathematics Placement Practice Test Solutions

The letter answer from the practice test is in parentheses after the solution.
The corresponding topics from the mathematics placement exam topic list are listed after that.

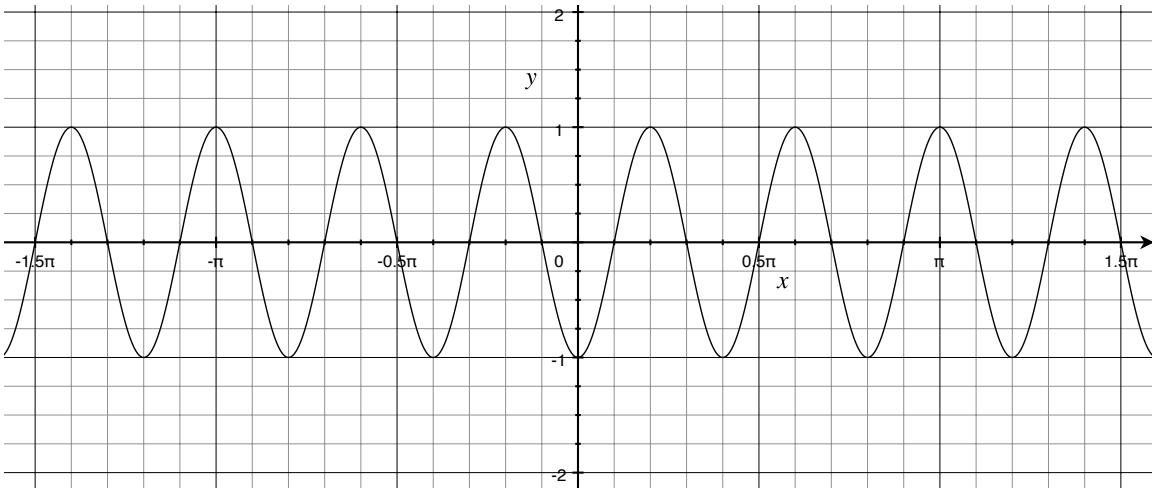
1. What is the y-coordinate of the intersection point of the two lines given by the equations:

   y=3x-2 and y=5x+6

   In order for the two lines to intersect, the x- and y-coordinates must be simultaneously equal. Because both equations are currently written in terms of y, we may set the other sides equal to each other:

   3x-2=5x+6 and solve for x:

   x=-4. Plugging back into either equation to solve for y, yields y=3(-4)-2=5(-4)+6=-14. Therefore, we report that the y-coordinate of the intersection point is -14. (d) Algebra 1,5
2. In a standard coordinate system, the graph of 7y+2x=15 is the same as the graph of the line written in y=mx+b form, which is y=(-2/7)x+(15/7). In this form, we see that the slope is -2/7. Because the slope is negative, the line falls to the right. (d) Algebra 2,3,4
3. A box has a square base. The height of the box is three times its width. If the volume of the box is 3000 cm^3, what is the width of the box?
   The volume of a box is length times width times height. In this case, length and width are equal, because of the square base. Call this length, s. The height is three times as big, so it is 3s. The total volume is then $3s^3=3000$. So $s^3=1000$ or s=10. The width of the box is then 10. (b) Algebra 10, 13, Geometry 4
4. The area of a triangle is 81 inches squared. Its base is half its height. What is the height of the triangle?
   Call the base of the triangle b and the height h. Then the area of the triangle is (1/2)bh. But b=(1/2)h. So the area is (1/2)(1/2)hh by substitution for b. Also, the area is given to be 81. Hence $81=(1/4)h^2$. That is, $h^2=4*81$. So h=18. (d) Algebra 10, 13, Geometry 2
5. Suppose that the number of cells in a petri dish is 600 one hour after the cells are plated into the dish. Three hours after being plated the cell population has grown to 3000. If the population of cells in the petri dish is growing exponentially, what is the number of cells in the dish 7 hours after they were plated?
   The general formula for an exponential function is $P(t)=Ca^t$, where P is the size of the function at time t, C is the size of the function at time 0, and a is some value greater than 0. We know that $600=Ca$ and $3000=Ca^3$. Dividing the last equation by the previous, we find that $a^2=3000/600 = 5$. Therefore, $P(7)=Ca^7=Ca^3(a^2)^2=3000(5)^2=3000(25)=75{,}000$. (d) Algebra 9, Pre-calculus 1,2
6. A function is said to have period B if f(x+B)=f(x) for all values of x, and B is the smallest such positive number. What is the period of the graph shown below?

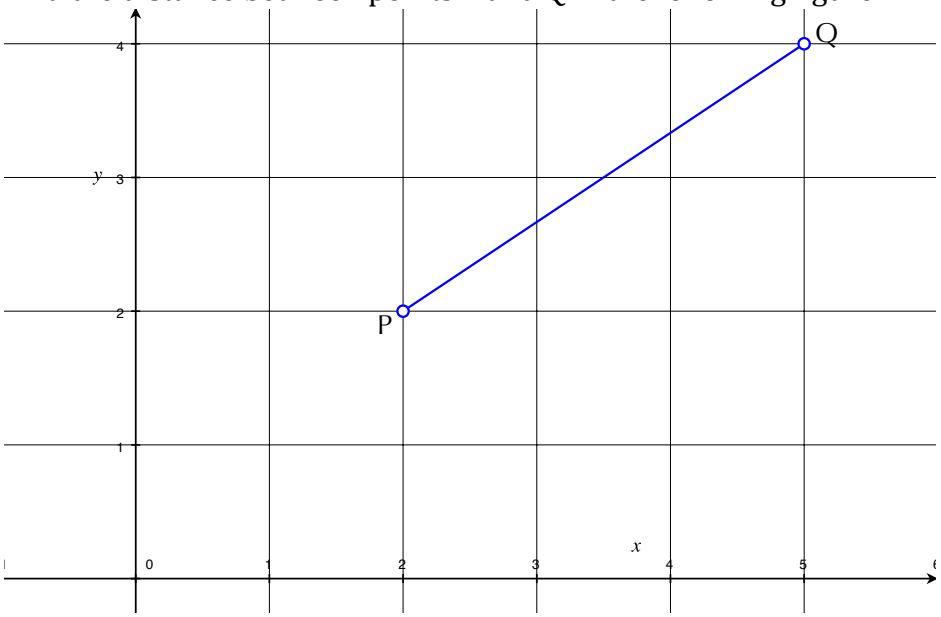


Examining the graph shows that it repeats a full cycle every 0.4π. (b) Trigonometry 1

7. The inequality $0 < |3x + 2| < d$ is equivalent to
   $0 \neq 3x + 2$ and $|3x + 2| < d$, which is equivalent to
   $x \neq \frac{-2}{3}$ and $-d < 3x + 2 < d$, which is equivalent to
   $x \neq \frac{-2}{3}$ and $-d - 2 < 3x < d - 2$, which is equivalent to
   $x \neq \frac{-2}{3}$ and $\frac{-d-2}{3} < x < \frac{d-2}{3}$. (d) Algebra 7

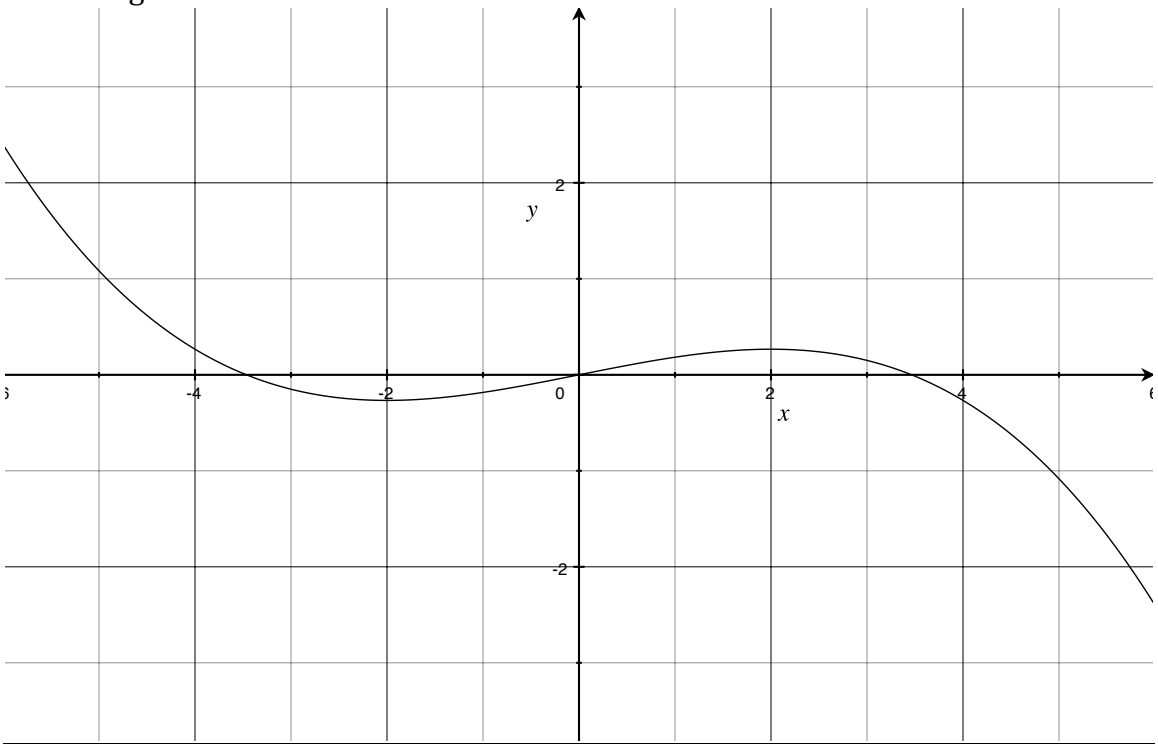
8. Find interval(s) on which $5x^2-9x+2 > 0$.
   The function $5x^2-9x+2$ is an upward facing parabola, so it will only be less than or equal to 0 between its zeros. For other x-values the function will be greater than 0. To find these zeros, we can use the quadratic formula:
   $x = \frac{9 \pm \sqrt{9^9 - 4 \times 5 \times 2}}{2 \times 5} = \frac{9 \pm \sqrt{81-40}}{10} = \frac{9 \pm \sqrt{41}}{10}$. (d) Algebra 6, 11

9. Find the distance between points P and Q in the following figure.



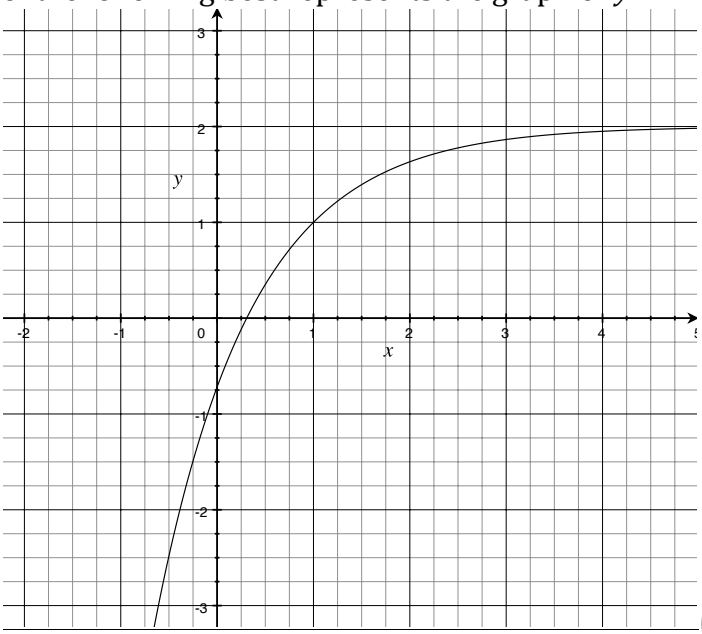
We can think of this as the distance formula or as the Pythagorean Theorem. The distance between P and Q=$\sqrt{(5-2)^2+(4-2)^2} = \sqrt{9+4} = \sqrt{13}$. (d) Algebra 8

10. At which value of of θ, $0 \leq \theta \leq \frac{\pi}{2}$, does csc(θ) = 2?
    Because csc(θ)=1/ sin(θ), this is the same question as: For what value of θ, $0 \leq \theta \leq \frac{\pi}{2}$, does sin(θ) = ½? The answer is $\frac{\pi}{6}$. (b) Trigonometry 2, 4
11. If $\log_3(2x+5)=4$, then x=
    $2x+5=3^4=81$, so $2x=81-5=76$. Hence, $x=76/2 = 38$. (d) Algebra 1, Pre-calculus 4,5
12. Let $g(t) = \sqrt{2t-3}$. Find $\frac{g(t+2)-g(t)}{2}$. Choose the most reduced answer.
    $\frac{g(t+2)-g(t)}{2} = \frac{\sqrt{2(t+2)-3}-\sqrt{2t-3}}{2} = \frac{\sqrt{2t+4-3}-\sqrt{2t-3}}{2} = \frac{\sqrt{2t-1}-\sqrt{2t-3}}{2}$. (d) Algebra 11, 12
13. A function, f, is increasing on an interval (a,b) if for x,y in (a,b) with x<y, f(x)<f(y). Determine the x-interval(s) on which the pictured graph is increasing.



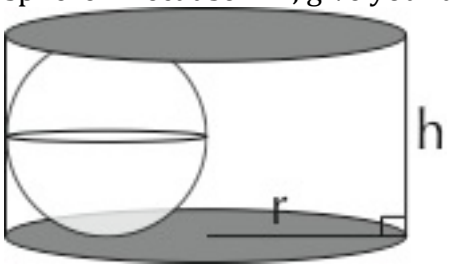
(-2,2) (b)

14. Which of the following best represents the graph of $y = 2 - e^{1-x}$?



(a) Algebra 12, Pre-calulus 6, 7

15. A cylinder in which the radius equals the height contains one largest possible sphere. How much volume is inside of the cylinder, but outside of the sphere? Because r=h, give your answer in terms of r.



The volume of a cylinder is $\pi r^2 h$, but because r=h, the volume of this cylinder is πr^3. The volume of the sphere is $(4/3)\pi R^3$, where R is the radius of the sphere. Because the sphere is the largest one that fits into the given cylinder, we know its diameter to equal r, so R=r/2. Therefore, the volume of the sphere is $(4/3)\pi(r/2)^3=(4/3)\pi r^3/8=\pi r^3/6$. The volume of the cylinder outside of the sphere is then $\pi r^3 - \pi r^3/6 = 5\pi r^3/6$. (d) Geometry 4,5