Mathematics Placement Exam Topic List

The mathematics placement exam emphasizes topics from algebra, geometry, pre-calculus, and trigonometry. For full understanding of the Mercer Math courses resting upon the knowledge from these subjects, please do not assume that you will only need to know the limited number of concepts inventoried below.

Some student learning objectives from Algebra, Geometry, Pre-calculus, and Trigonometry that might be tested on the Mathematics Placement Exam are listed below. This list does not promise to be comprehensive. Others may categorize the objectives into different courses.

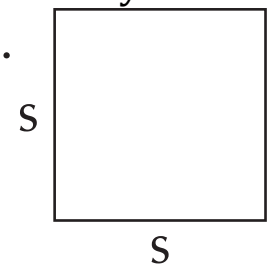
Please note that formulas are not given on the placement test. They are to be memorized or internalized.
For each of the following, students will know how and be able to accurately:

Algebra

1. Solve an equation to find the value of a variable.
2. Graph the equation of a line.
3. Find the slope of a line.
4. Explain how the slope of a line relates to the appearance of a line.
    a. A line with zero slope is horizontal (e.g., y=7)
    b. A line with infinite slope is vertical (e.g., x=2)
    c. A line with positive slope could be described as increasing as one moves to the right or it could be described as rising to the right (e.g., y-3=5(x-4) or y=5x-17 or 2y-10x+34=0.)
    d. A line with negative slope could be described as decreasing as one moves to the right or it could be described as falling to the right (e.g., y-1=-2(x+1) or y=-2x+3 or 5y+10x=15.)
5. Find the intersection of two lines.
    a. Report the intersection point.
    b. Report only the x-coordinate of the intersection.
    c. Report only the y-coordinate of the intersection.
6. Solve inequalities involving quadratic functions.
7. Solve an inequality with absolute values.
8. Find the distance between two points in the Cartesian plane.
9. Simplify or expand an expression with exponents, including fractional and negative exponents.
10. Understand the role of exponents in working with numbers./Have number sense when it comes to exponents.
11. Factor algebraic expressions.
12. Evaluate a function at a given value.
13. Read, interpret, set up, and solve a basic word problem.

Geometry

A.



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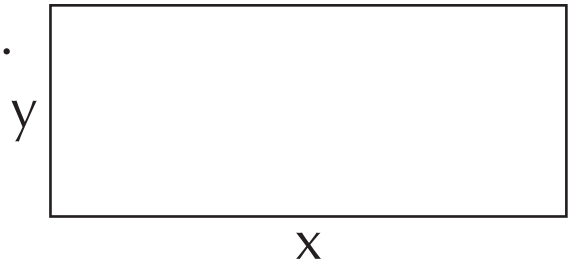

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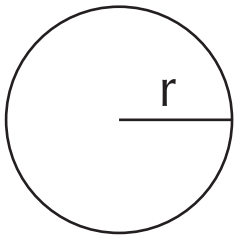

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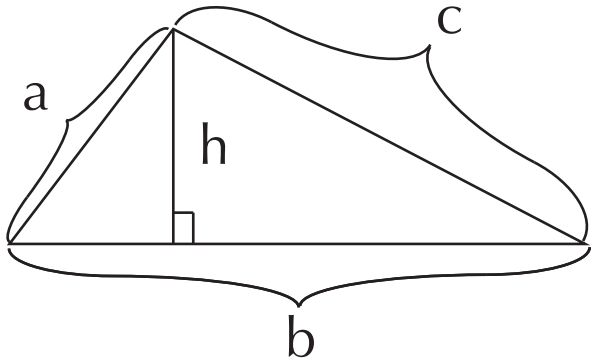
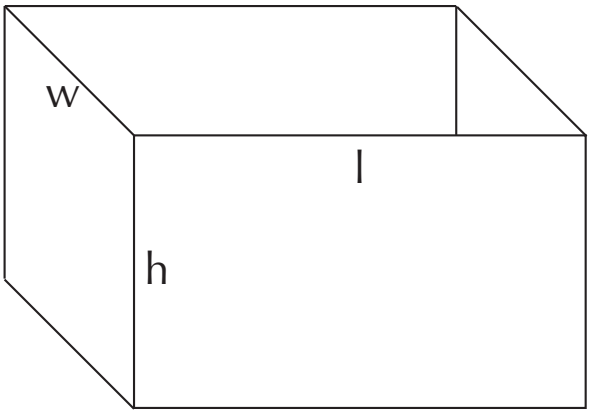


1. Use the formulas for perimeters of standard shapes.
    a. A. Square: perimeter=4s
    b. B. Rectangle: perimeter=2x+2y
    c. C. Circle: perimeter= $2\pi r$
    d. D. Triangle: perimeter=a+b+c
2. Use the formulas for areas of standard shapes.
    a. A. Square: area= s^2
    b. B. Rectangle: area=xy
    c. C. Circle: area=πr^2
    d. D. Triangle: area=(1/2)bh

E.



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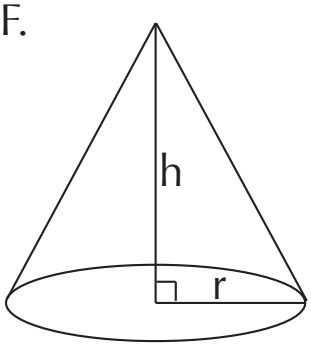

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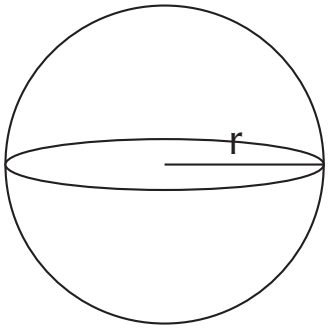

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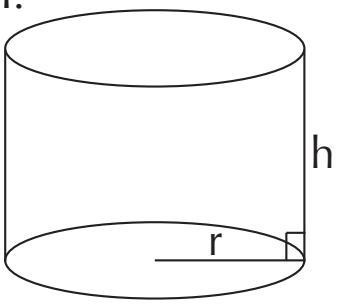


3. Use the formulas for surface areas of standard shapes.
    a. E. Box with all 6 sides: Surface Area (SA) = 2lw + 2wh + 2lh
    b. E. Box with open top, but closed bottom:
    Surface Area (SA) = lw +2wh + 2lh
    c. G. Sphere: Surface Area (SA) = $4\pi r^2$
    d. H. Cylinder with both bases: Surface Area (SA) = $2\pi r^2+2\pi rh$
    e. H. Cylindrical side, no bases: Surface Area (SA)= $2\pi rh$
    f. Students are not expected to know the surface area of a cone given only the height and base radius.
4. Use the formulas for volumes of standard shapes.
    a. E. Box: Volume = lwh
    b. F. Cone: Volume = $(1/3)\pi r^2h$
    c. G. Sphere: Volume = $(4/3)\pi r^3$
    d. H. Cylinder: Volume = πr^2h
5. Reason using geometric logic, such as by using proportions.

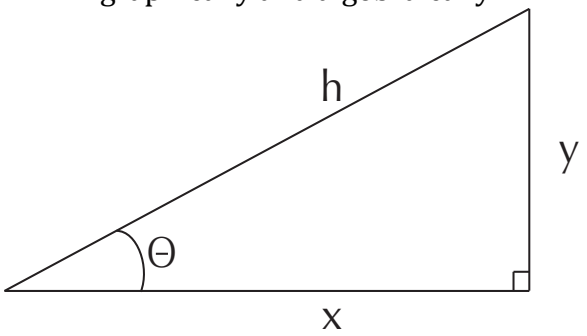
Pre-Calculus

1. Recognize the use of or employ the laws of exponents. For c,d,g real numbers and a,b > 0, x,y variables taking on real values,
    a. $c^{d+g}=c^dc^g$ and $a^{x+y}=a^xa^y$
    b. $c^{d-g}=\frac{c^d}{c^g}$ and $a^{x-y}=\frac{a^x}{a^y}$
    c. $(c^d)^g=c^{dg}$ and $(a^x)^y=a^{xy}$
    d. $c^gd^g=(cd)^g$ and $a^xb^x=(ab)^x$

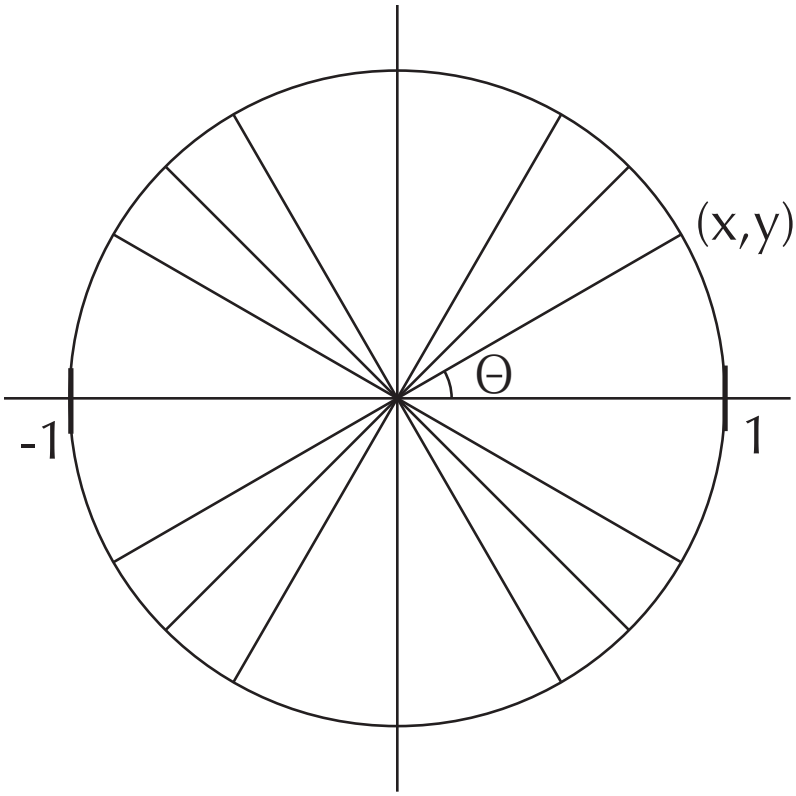
2. Solve exponential growth problems, which might be presented as word problems. Contexts may include banking, population growth or decay, or radioactive decay.
3. Recognize the use of or employ the laws of logarithms. For c,d,g positive real numbers and a,b > 0, x,y variables taking on positive real values,
    a. $\log_c(dg) = \log_c d + \log_c g$ and $\log_a(xy) = \log_a x + \log_a y$
    b. $\log_c(d/g) = \log_c d - \log_c g$ and $\log_a(x/y) = \log_a x - \log_a y$
    c. $\log_c(d^g) = g(\log_c d)$ and $\log_a(x^b) = b(\log_a x)$
4. Realize that logarithms and exponential functions in the same base are inverse functions and exploit that to simplify and solve equations. (I.e., $\log_a(a^x)=x$.)
5. Solve logarithmic equations, which might be presented as word problems. These problems are often part of what originally seem to be exponential growth problems.
6. Recognize and sketch the graphs of standard functions such as lines, power functions (x^2, x^3, x^{-1}, x^{-2}, x^n, etc. and multiples of these), exponential functions, and logarithmic functions, all with variations.
7. Understand the role of transformations on graphs and determine the graph of a function as the transformation of a standard graph using the algebraic expression of the function.
8. Solve for the zeros of polynomials and rational functions.
9. Find the domain of rational and root functions.

Trigonometry

1. Understand the meanings of period and amplitude of a function both graphically and algebraically.



2. Use the six trigonometric (trig) functions via their definitions:
    a. sine: $\sin(\theta) = \frac{y}{h}$
    b. cosine: $\cos(\theta) = \frac{x}{h}$
    c. tangent: $\tan(\theta) = \frac{y}{x}$
    d. cosecant: $\csc(\theta) = \frac{h}{y}$
    e. secant: $\sec(\theta) = \frac{h}{x}$
    f. cotangent: $\cot(\theta) = \frac{x}{y}$
3. Use the definitions of the trig functions to reduce trigonometric expressions



und=undefined; deg=degrees, rad=radians

Θ deg	**Θ rad**	**x**	**y**	**sin(θ)**	**cos(θ)**	tan(θ)	csc(θ)	sec(θ)	cot(θ)
0	**0**	**1**	**0**	**0**	**1**	0	und	1	und
30	**π/6**	$\frac{\sqrt{3}}{2}$	$\frac{1}{2}$	$\frac{1}{2}$	$\frac{\sqrt{3}}{2}$	$\frac{1}{\sqrt{3}}$	2	$\frac{2}{\sqrt{3}}$	$\sqrt{3}$
45	**π/4**	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{2}}{2}$	1	$\sqrt{2}$	$\sqrt{2}$	1
60	**π/3**	$\frac{1}{2}$	$\frac{\sqrt{3}}{2}$	$\frac{\sqrt{3}}{2}$	$\frac{1}{2}$	$\sqrt{3}$	$\frac{2}{\sqrt{3}}$	2	$\frac{1}{\sqrt{3}}$
90	**π/2**	**0**	**1**	**1**	**0**	und	1	und	0
120	2π/3	$\frac{-1}{2}$	$\frac{\sqrt{3}}{2}$	$\frac{\sqrt{3}}{2}$	$\frac{-1}{2}$	$-\sqrt{3}$	$\frac{2}{\sqrt{3}}$	-2	$\frac{-1}{\sqrt{3}}$
135	3π/4	$\frac{-\sqrt{2}}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{-\sqrt{2}}{2}$	-1	$\sqrt{2}$	$-\sqrt{2}$	-1
150	5π/6	$\frac{-\sqrt{3}}{2}$	$\frac{1}{2}$	$\frac{1}{2}$	$\frac{-\sqrt{3}}{2}$	$\frac{-1}{\sqrt{3}}$	2	$\frac{-2}{\sqrt{3}}$	$-\sqrt{3}$
180	π	-1	0	0	-1	0	und	-1	und
210	7π/6	$\frac{-\sqrt{3}}{2}$	$\frac{-1}{2}$	$\frac{-1}{2}$	$\frac{-\sqrt{3}}{2}$	$\frac{1}{\sqrt{3}}$	-2	$\frac{-2}{\sqrt{3}}$	$\sqrt{3}$

225	5π/4	$\frac{-\sqrt{2}}{2}$	$\frac{-\sqrt{2}}{2}$	$\frac{-\sqrt{2}}{2}$	$\frac{-\sqrt{2}}{2}$	1	$-\sqrt{2}$	$-\sqrt{2}$	1
240	4π/3	$\frac{-1}{2}$	$\frac{-\sqrt{3}}{2}$	$\frac{-\sqrt{3}}{2}$	$\frac{-1}{2}$	$\sqrt{3}$	$\frac{-2}{\sqrt{3}}$	-2	$\frac{1}{\sqrt{3}}$
270	3π/2	0	-1	-1	0	Und	-1	und	0
300	5π/3	$\frac{1}{2}$	$\frac{-\sqrt{3}}{2}$	$\frac{-\sqrt{3}}{2}$	$\frac{1}{2}$	$-\sqrt{3}$	$\frac{-2}{\sqrt{3}}$	2	$\frac{-1}{\sqrt{3}}$
315	7π/4	$\frac{\sqrt{2}}{2}$	$\frac{-\sqrt{2}}{2}$	$\frac{-\sqrt{2}}{2}$	$\frac{\sqrt{2}}{2}$	-1	$-\sqrt{2}$	$\sqrt{2}$	-1
330	11π/6	$\frac{\sqrt{3}}{2}$	$\frac{-1}{2}$	$\frac{-1}{2}$	$\frac{\sqrt{3}}{2}$	$\frac{-1}{\sqrt{3}}$	-2	$\frac{2}{\sqrt{3}}$	$-\sqrt{3}$
360	2π	1	0	0	1	0	und	1	Und
-30	-π/6 =11π/6	$\frac{\sqrt{3}}{2}$	$\frac{-1}{2}$	$\frac{-1}{2}$	$\frac{\sqrt{3}}{2}$		-2	$\frac{2}{\sqrt{3}}$	$-\sqrt{3}$
etc.									

4. Use values of the trig functions at the standard angles (in radians) and their multiples to evaluate trigonometric expressions without a calculator. (Students should be able to find the value of any of the six trig functions at any standard angle based on having memorized the value of the functions sin(θ) and cos(θ) at x=0, x=π/6, x=π/4, x=π/3, and x=π/2. These are the bolded entries in the table.)
5. Graph the six trigonometric functions and their algebraic transformations.
6. Use the double and half angle fomulas for sine and cosine.
    a. $\sin(2\theta) = 2\sin(\theta)\cos(\theta)$
    b. $\cos(2\theta) = \cos^2(\theta)-\sin^2(\theta) = 2\cos^2(\theta) - 1 = 1 - 2\sin^2(\theta)$
    c. $\sin^2(\theta) = \frac{1-\cos(2\theta)}{2}$
    d. $\cos^2(\theta) = \frac{1+\cos(2\theta)}{2}$
7. Use the main trig identities and their variants that arise from moving terms from one side of the equality to the other through subtraction.
    a. $\sin^2\theta + \cos^2\theta = 1$
    b. $\tan^2\theta + 1 = \sec^2\theta$
    c. $1 + \cot^2\theta = \csc^2\theta$
    d. $\sin(-\theta) = -\sin(\theta)$
    e. $\cos(-\theta) = \cos(\theta)$
    f. $\tan(-\theta) = -\tan(\theta)$
    g. $\sin\left(\frac{\pi}{2} - \theta\right) = \cos(\theta)$
    h. $\cos\left(\frac{\pi}{2} - \theta\right) = \sin(\theta)$